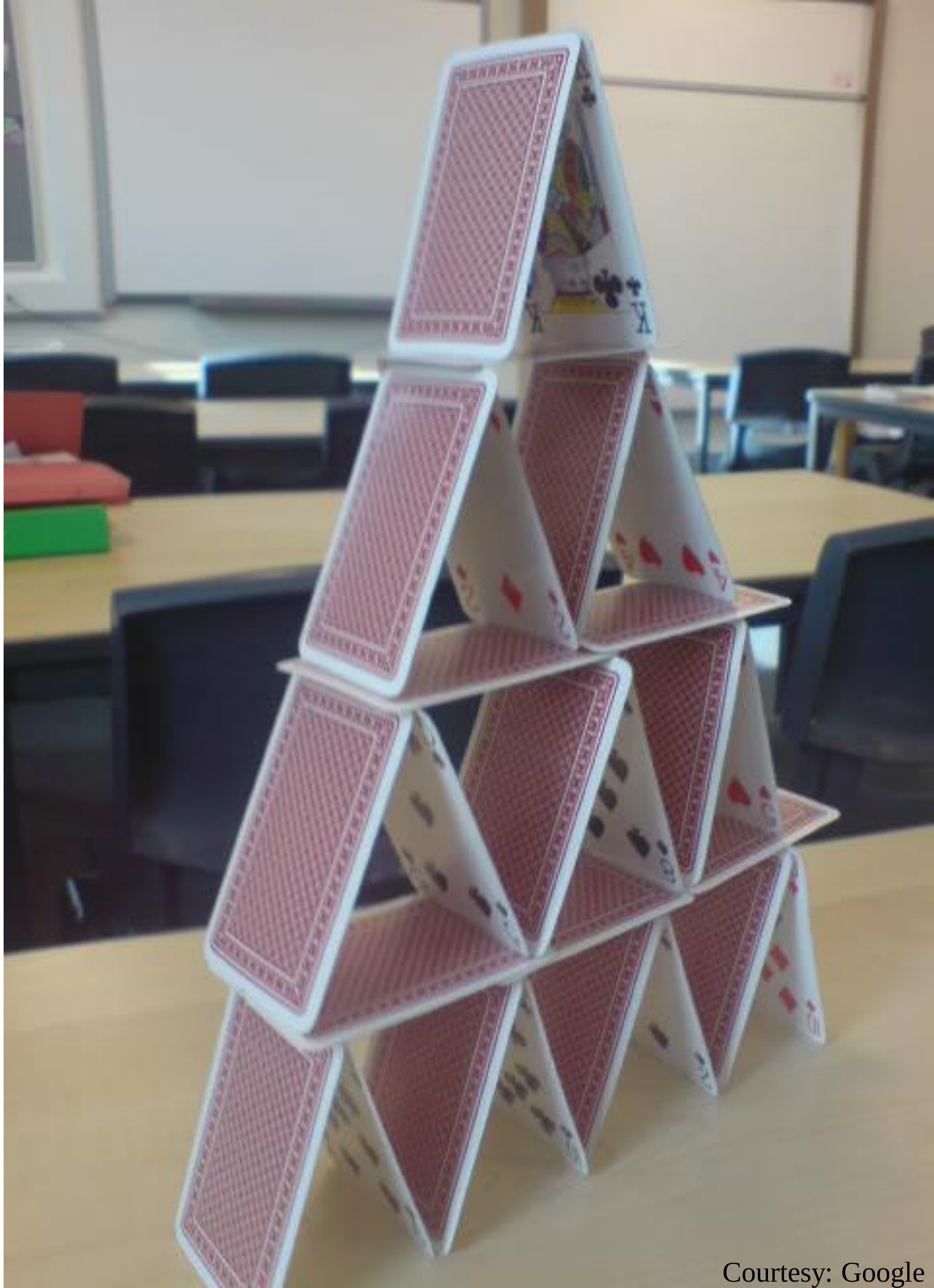
Mathematics
&
Tower of cards

In mathematics we use abstraction to comprehend nature and its working. That abstraction rely on the structure of logical and analytical reasoning just like a tower of cards. Each card is connected with others in a way if we disturb any one the whole structure collapses.

Everything that is built on it must obey all basic and fundamental rules such that it can be proved analytically.



Courtesy: Google

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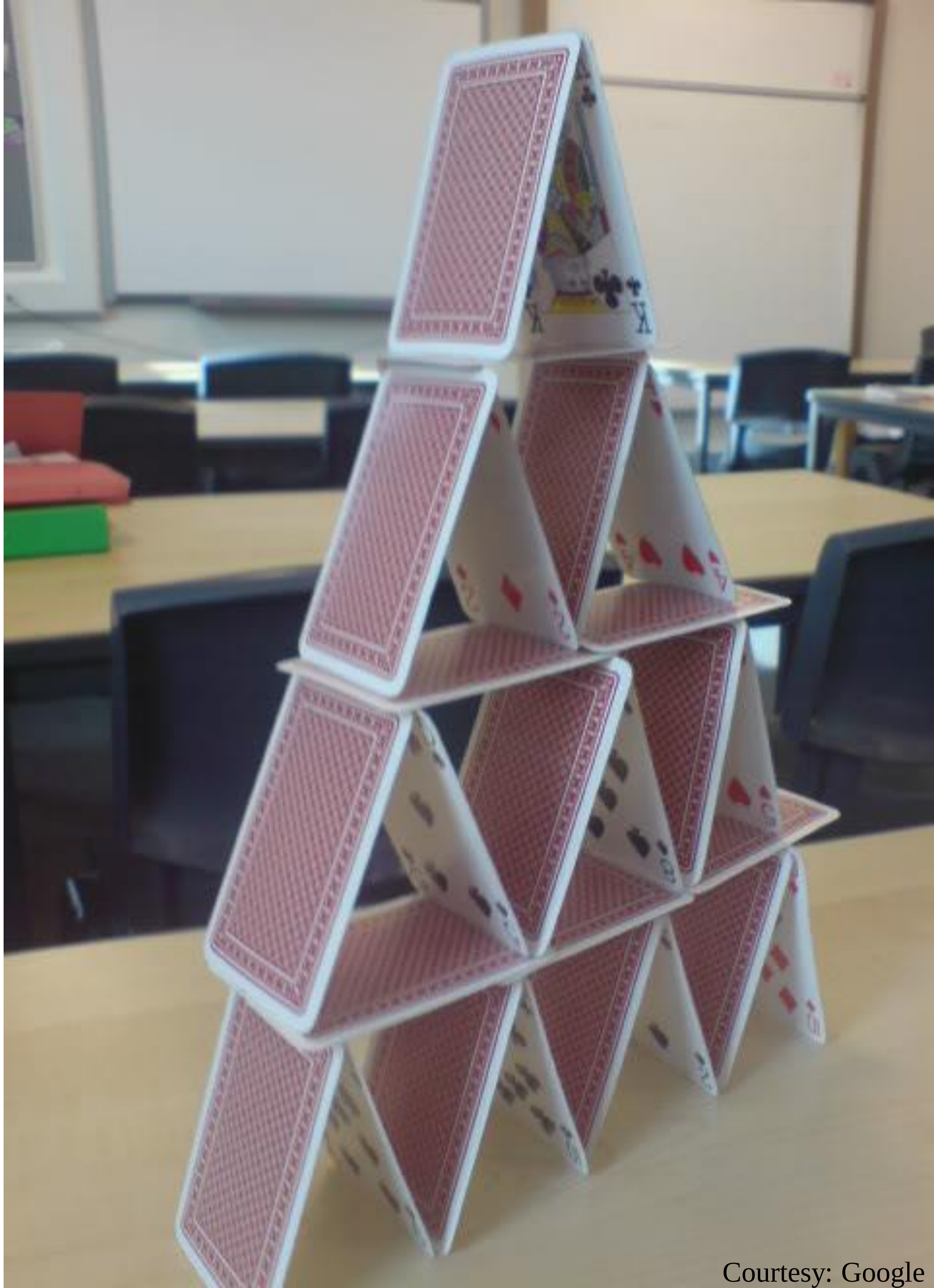
Therefore it is important for us to nurture our abilities to prove or disproof on the basis of logical and critical reasoning. This is how science (of which engineering occupies only a fractional part) should work.

It is very common now a days to hear about false flags or claims for example: speed of an object is faster than light; someone had a time travel last night; Einstein's theory of relativity is false etc..



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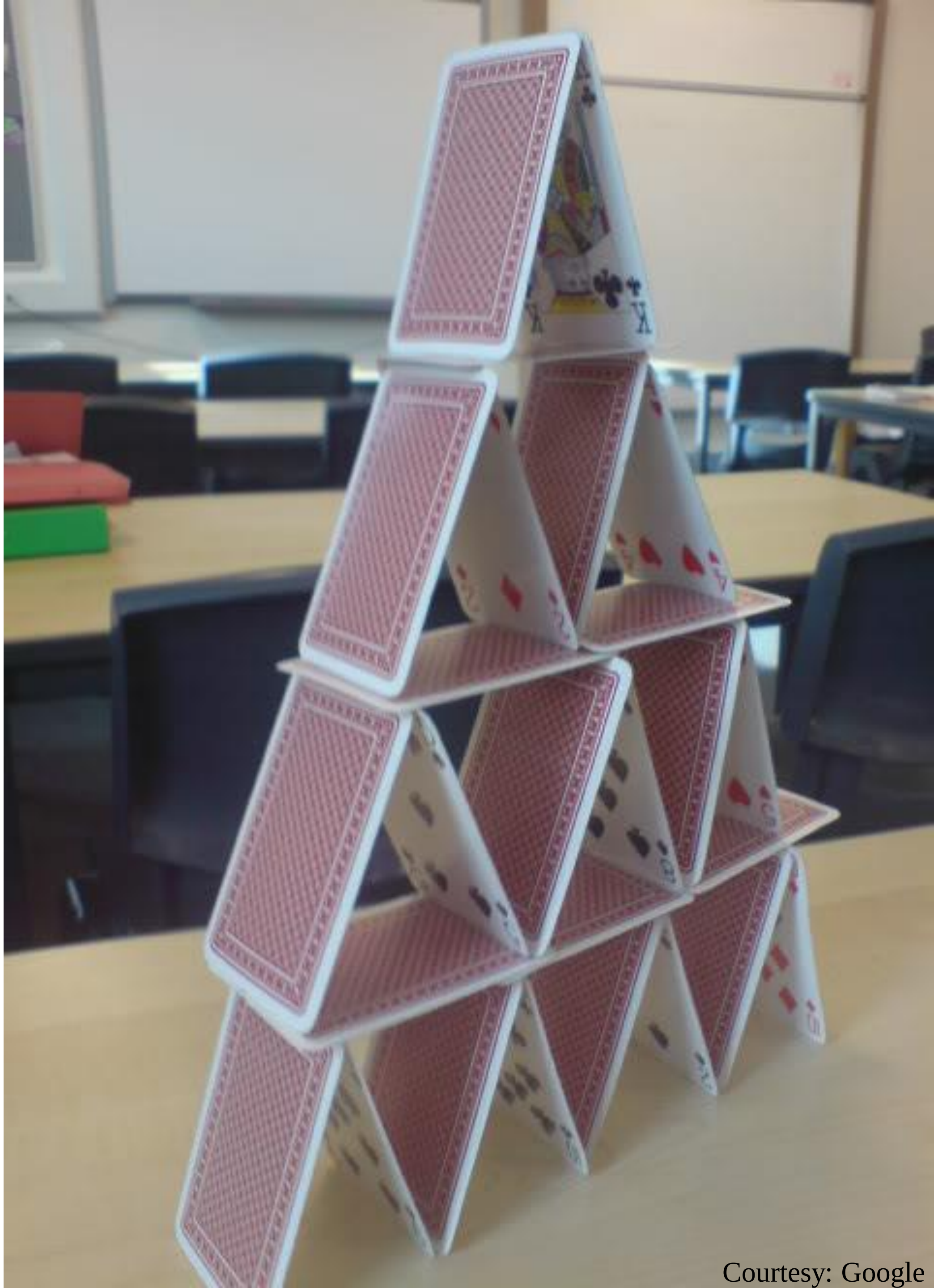
Such claims can be disproved easily using basic logic and reasoning on which the original theory or idea is built.

It is also not true that we can not criticize or touch the fundamental concepts. On the contrary it is always possible to find a more fundamental structure which means to add one more layer of cards (containing more cards) into the base of the tower of cards.



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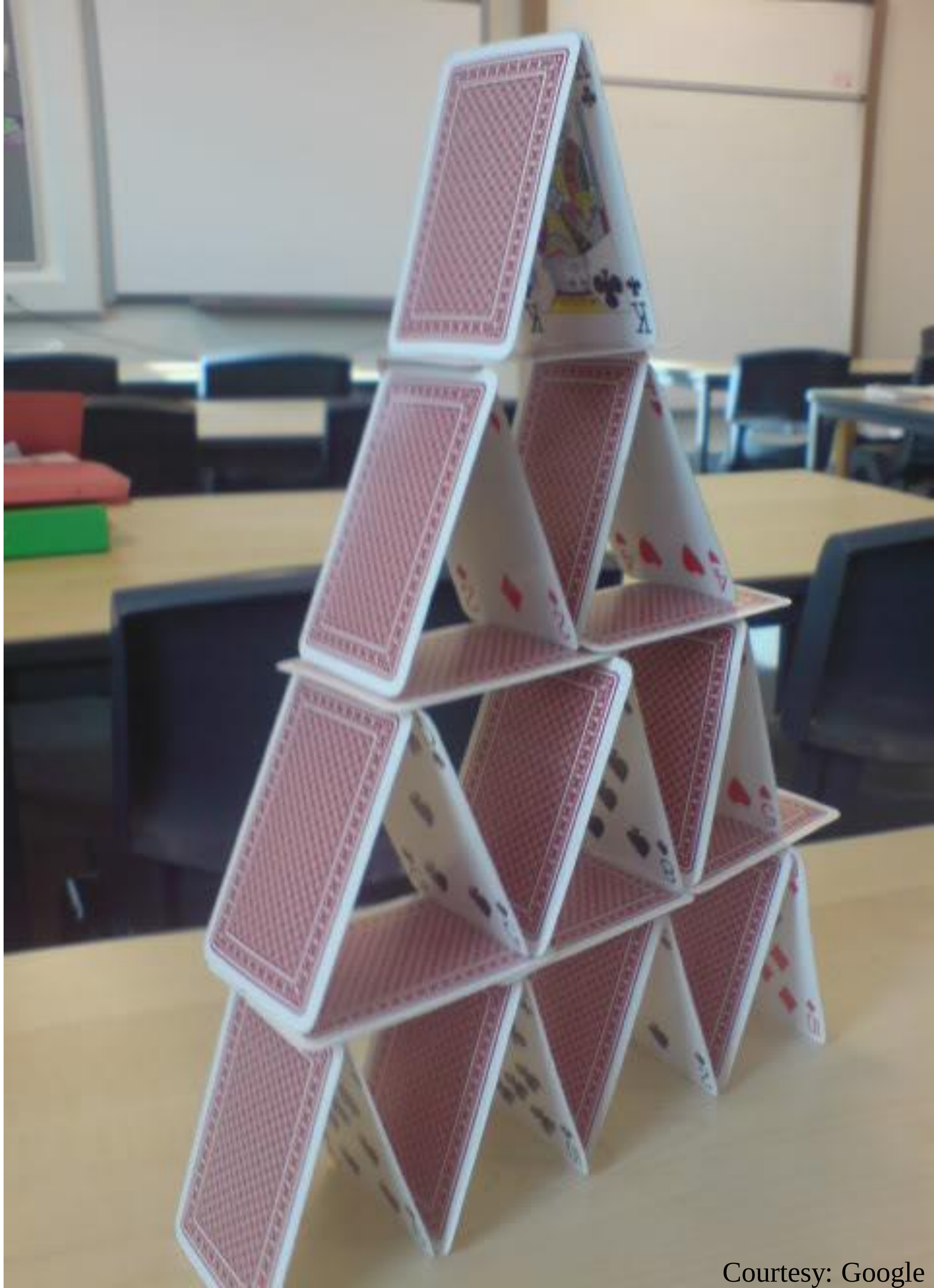
For example, Einstein brought a new layer (by introducing spacetime which is relative) at the foundation of the structure made by Newton.



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Applications of Triple Integration

- Mass of a rigid body
- Center of mass of a rigid body
- Moment of inertia

Center of Mass

1D

For a system of two particles of masses m_1 and m_2 located at x_1 and x_2 the center of mass is a point

$$X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

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- It is a point where whole mass of the system is concentrated and motion of the system can be described as if the motion of a particle at that point.
- It helps us in understanding the mechanics of system of particles.
- A key fact while calculating the motions of planets in Kepler's time as if the whole mass of planet is concentrated at a single point.

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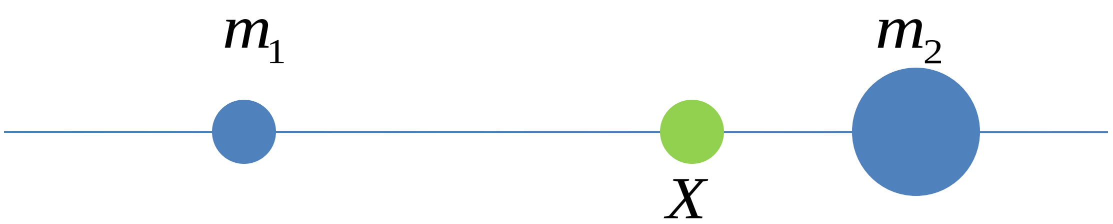

$m_1 < m_2$

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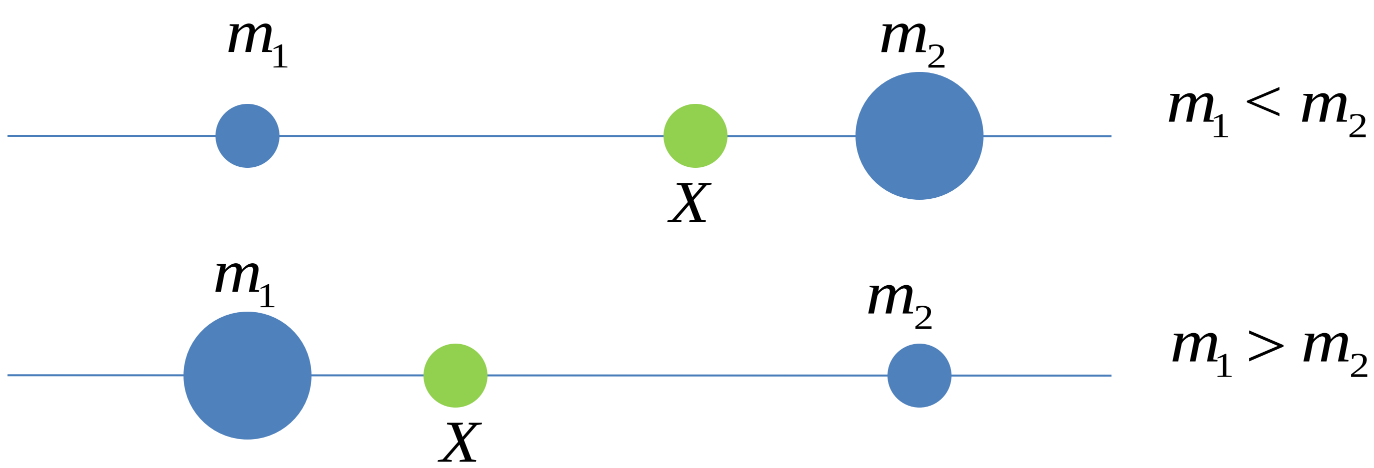

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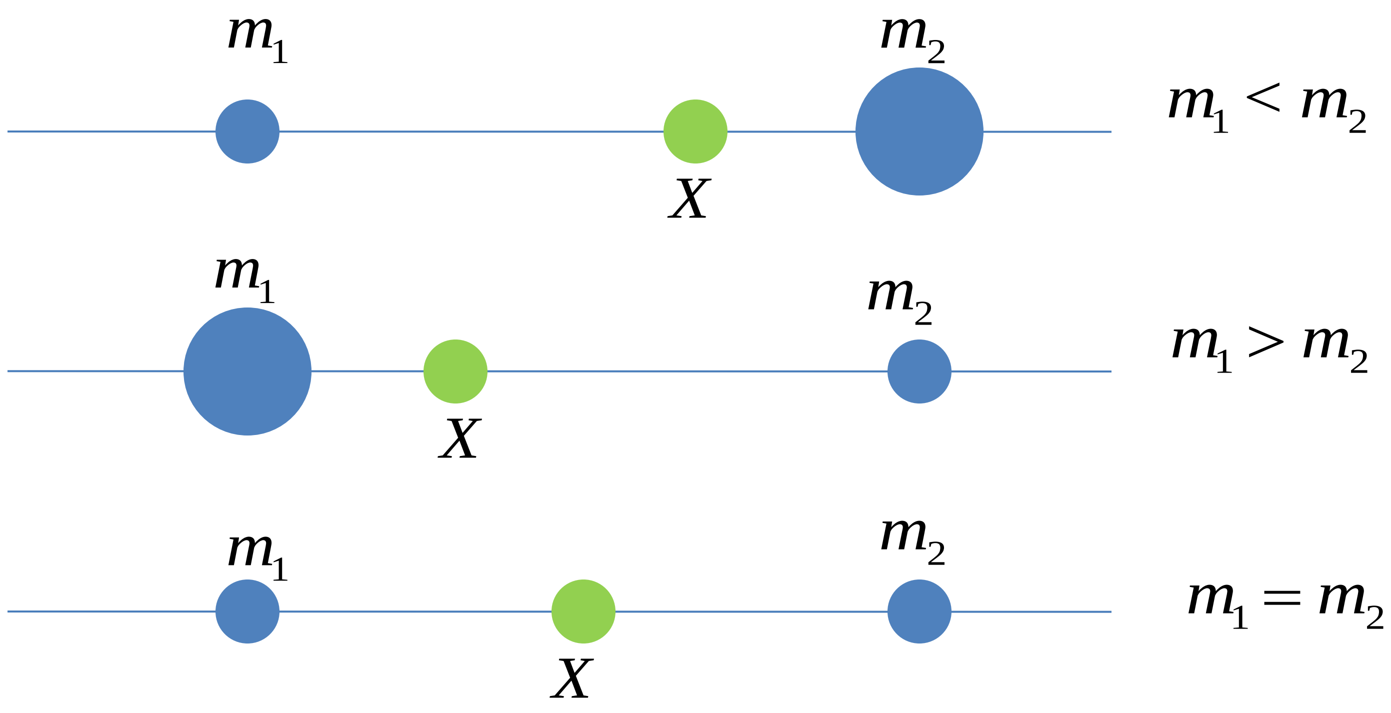

Center of Mass (Example) 1D

For a system of two particles of masses $2kg$ and $5kg$ located at $1m$ and $4m$ from origin on the x - axis then the center of mass is

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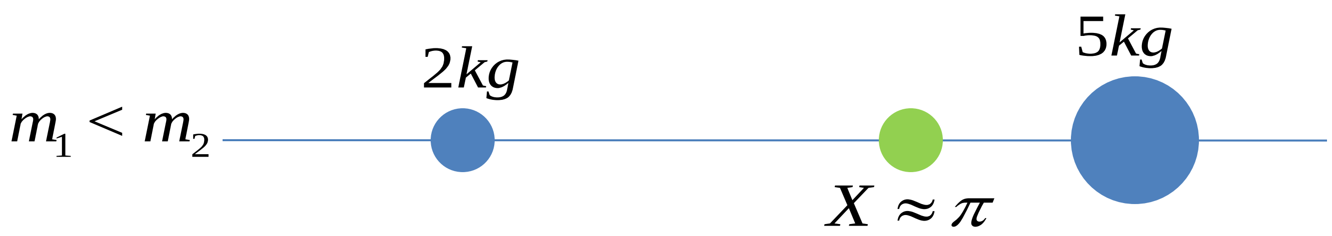


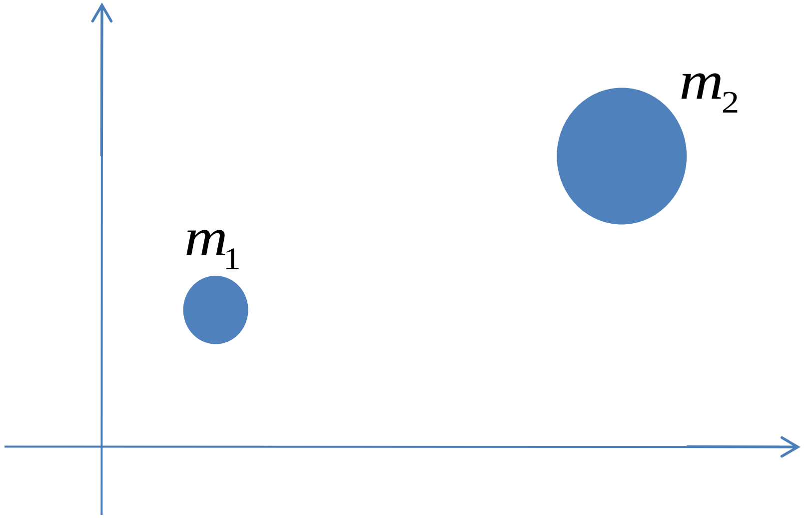
Physical Interpretation

π

Center of Mass

2D

For a system of two particles of masses m_1 and m_2 located at (x_1, y_1) and (x_2, y_2) the center of mass is a point



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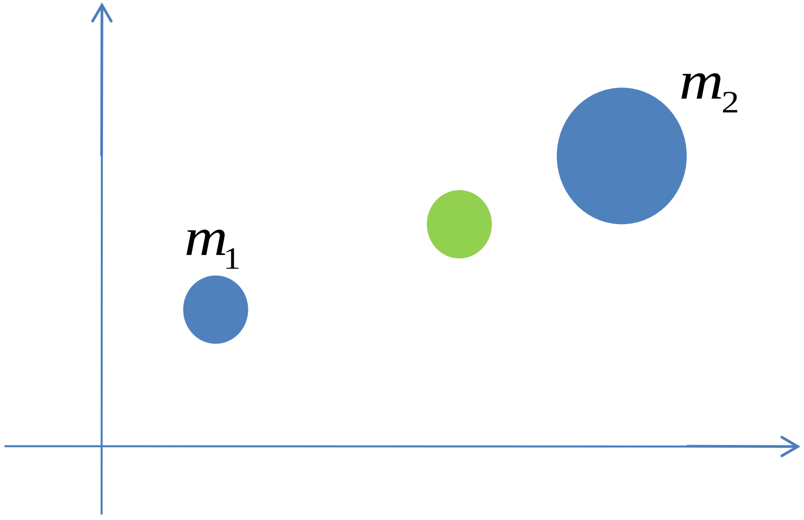
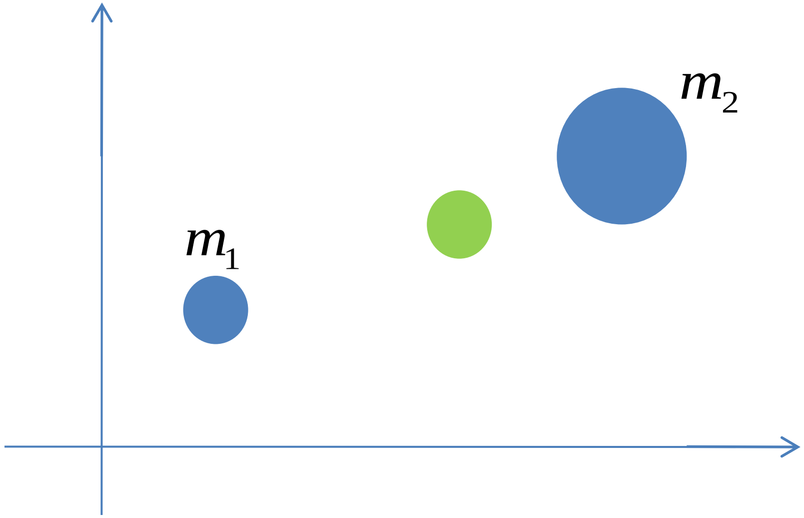


Center of Mass

2D

For a system of two particles of masses m_1 and m_2 located at (x_1, y_1) and (x_2, y_2) the center of mass is a point

$$X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \qquad Y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$



Center of Mass (Example) 2D

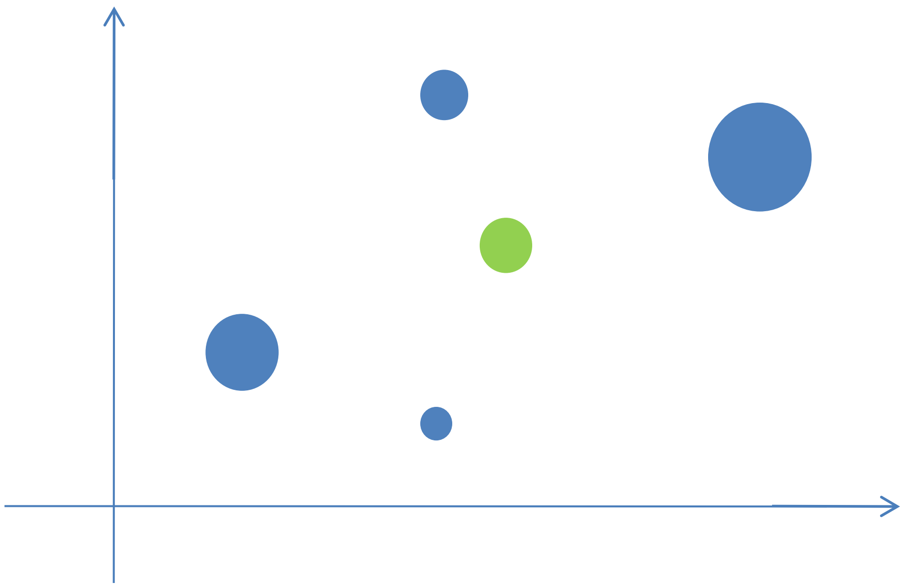
Construct examples own your own and find another 2D physical interpretation of π.

Center of Mass

2D

For a system of 'n' particles of masses $m_1, m_2, \ldots, m_n$ located at $(x_i, y_i),\ 1 \le i \le n$ the center of mass is a point

$$X = \frac{m_1 x_1 + m_2 x_2 + \ldots + m_n x_n}{m_1 + m_2 + \ldots + m_n} \qquad Y = \frac{m_1 y_1 + m_2 y_2 + \ldots + m_n y_n}{m_1 + m_2 + \ldots + m_n}$$



Center of Mass of a Planar Region: 2D

A planar region is composed of many particles and in order to calculate its center of mass one has to depart from *discrete to continuum* in which case we would use the idea of integration.

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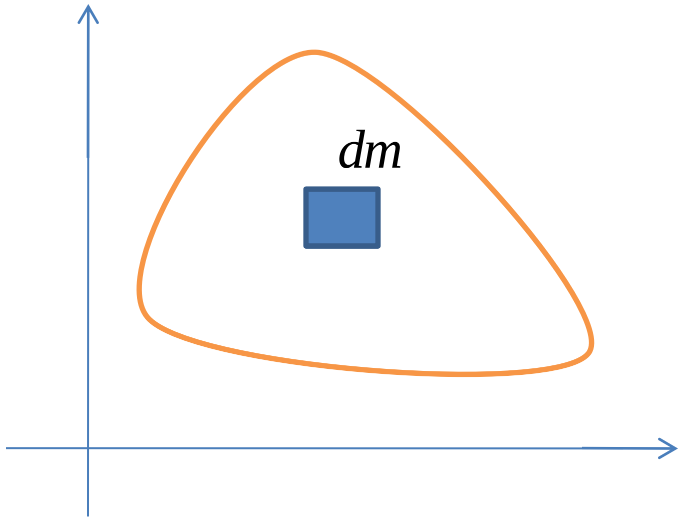


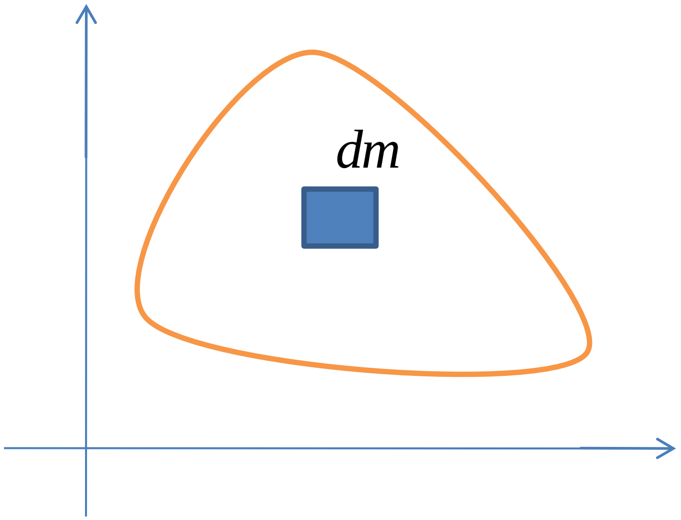
Center of Mass of a Planar Region: **2D**

A planar region is composed of many particles and in order to calculate its center of mass one has to depart from *discrete to continuum* in which case we would use the idea of integration.

M = total mass of the rigid body

$$\sum \dashrightarrow \iint$$

$$M = \int dm$$



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M = total mass of the rigid body

$$\sum \dashrightarrow \iint$$

$$M = \int dm$$

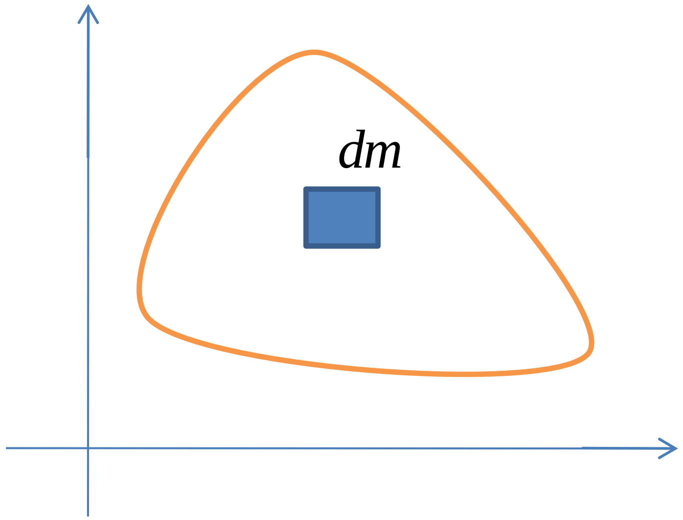
How to compute integral?

$$\text{Since } \sigma = \frac{m}{A}$$

$$\Rightarrow m = \sigma A$$

$$\Rightarrow dm = \sigma dA$$

$$\Rightarrow dm = \sigma dx dy$$

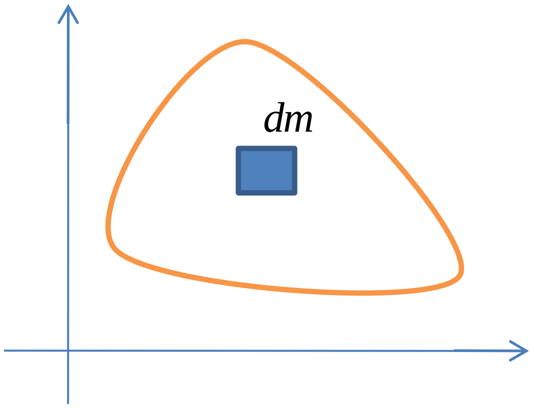


Strong assumption is made that σ is constant.

Center of Mass of a Planar Region: 2D

$$\Rightarrow dm = \sigma dx dy \qquad \text{mass element}$$

$$\Rightarrow M = \int dm = \iint \sigma dx dy$$



Center of Mass of a Planar Region: **2D**

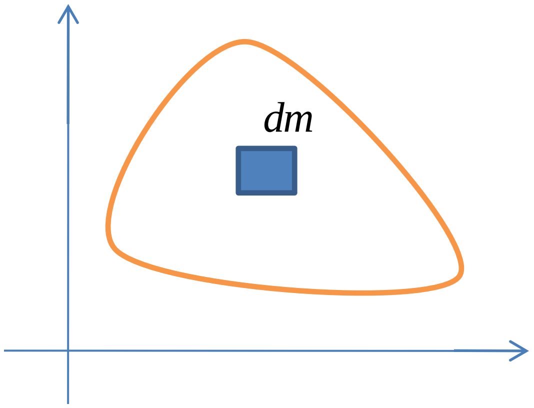
$$\Rightarrow dm = \sigma dxdy \qquad \text{mass element}$$

$$\Rightarrow M = \int dm = \iint \sigma dxdy$$

Therefore the center of mass is

$$X = \frac{\iint x\, dm}{M} = \frac{\iint x\sigma\, dxdy}{\iint \sigma\, dxdy}$$

$$Y = \frac{\iint y\, dm}{M} = \frac{\iint y\sigma\, dxdy}{\iint \sigma\, dxdy}$$



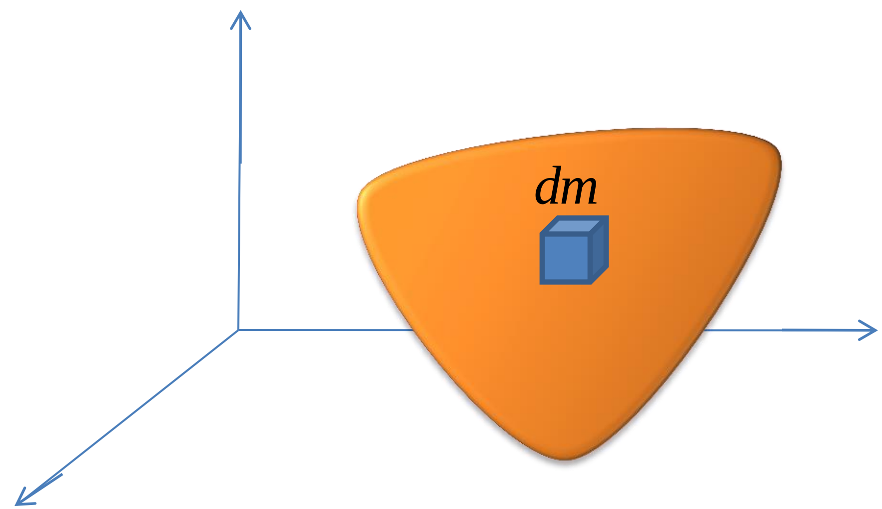
Center of Mass of a Rigid Body: **2D**

A rigid body is also composed of many particles and in order to calculate its center of mass one has to depart from *discrete to continuum* in which case we would use the idea of integration.

M = Total mass of the rigid body

$$\sum \dashrightarrow \iiint$$

$$M = \int dm$$



How to compute integral?

$$\text{Since } \rho = \frac{m}{V}$$

$$\Rightarrow m = \rho V$$

$$\Rightarrow dm = \rho\, dV$$

$$\Rightarrow dm = \rho\, dxdydz$$

Strong assumption is made that ρ is constant.

Center of Mass of a Rigid Body:

2D

$$\Rightarrow dm = \rho\, dxdydz \qquad \text{mass element}$$

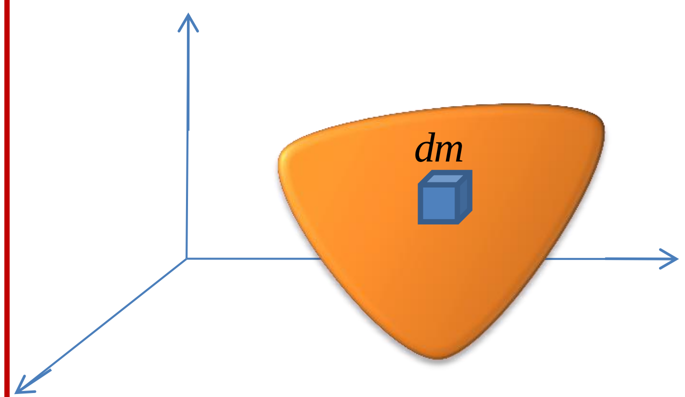
$$\Rightarrow M = \int dm = \iiint \rho\, dxdydz$$

Therefore the center of mass is

$$X = \frac{\iiint x\, dm}{M} = \frac{\iiint x \rho\, dxdydz}{\iiint \rho\, dxdydz}$$

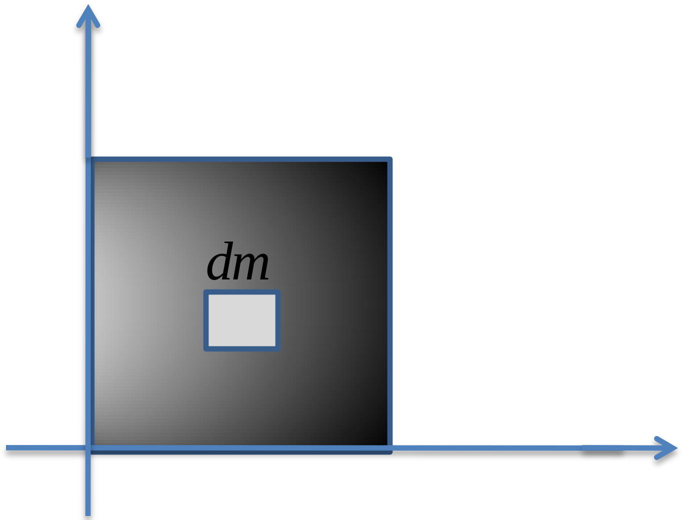
$$Y = \frac{\iiint y\, dm}{M} = \frac{\iiint y \rho\, dxdydz}{\iiint \rho\, dxdydz}$$

$$Z = \frac{\iiint z\, dm}{M} = \frac{\iiint z \rho\, dxdydz}{\iiint \rho\, dxdydz}$$



Center of Mass (Example) 2D

Q. Find the center of mass of a square of length 1 whose density is varying linearly along horizontal axis.



Center of Mass (Example) 2D

Q. Find the center of mass of a square of length 1 whose density is varying linearly along horizontal axis.

Ans. Since the density varies linearly therefore

$$\sigma(x,y) = \alpha x,\ 0 \le x \le 1$$

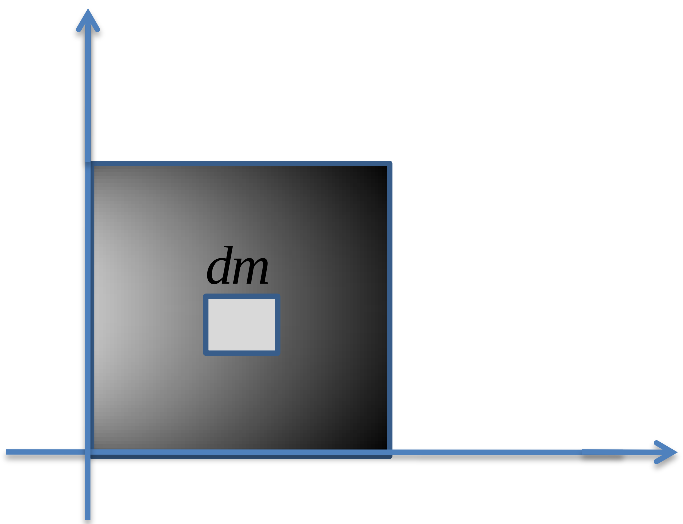
The total mass of the object is

$$M = \int dm = \iint \sigma dxdy$$

$$= \int_0^1 \int_0^1 \alpha x\, dxdy = \alpha \int_0^1 \tfrac{1}{2}\, dy = \frac{\alpha}{2}$$

$$X = \frac{\iint x\, dm}{M} = \frac{\iint \alpha x^2 dxdy}{\alpha / 2} = \frac{\alpha / 3}{\alpha / 2} = \frac{2}{3}$$

Similarly $Y = 2/3$



Thus the center of mass of the object is shifted from the geometric center because the density varies. It is nearly (0.67,0.67) more closer to the left hand side.

Center of Mass (Example) 2D

Q. Find the center of mass of a square of length 2π whose density is varying sinusoidaly.

Ans. Since the density varies sinusoidaly therefore

$$\sigma(x, y) = \sin x, \; -\pi \le x \le \pi$$

Calculate yourself …